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Soft Intersection-gamma Product of Groups

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ABSTRACT

Soft set theory offers an algebraic framework for modeling systems characterized by ambiguity, uncertainty, and parameter-dependent variability. This study introduces the soft intersection–gamma product, a new binary operation on soft sets whose parameter domains follow a group-theoretic structure. The operation is defined axiomatically and shown to be fully compatible with extended notions of soft equality and soft subsethood. We also examine the operation with respect to identity, absorbing, null, and absolute soft sets. Its structural properties, including closure, associativity, commutativity, idempotency, and distributivity, are studied in detail. The results demonstrate that the operation satisfies all algebraic constraints of group-indexed domains, thereby forming a robust and coherent algebraic system on the universe of soft sets. Beyond its theoretical significance, the operation reinforces the algebraic basis of soft set theory and provides a framework for developing a generalized soft group theory. Furthermore, its coherence with soft subset and equality relations increases its applicability in classification, decision-making, and uncertainty-aware modeling. These connections highlight its potential for both theoretical development and practical use.

Contents

1. Introduction

Several mathematically precise frameworks have been developed to address ambiguity, indeterminacy, and uncertainty across diverse domains, including engineering, economics, the social sciences, and medical diagnostics. However, basic models such as fuzzy set theory and probabilistic systems [1] have intrinsic limitations. Fuzzy sets rely on subjective membership functions, whereas probabilistic models presuppose accurate distributions and reproducible conditions, assumptions that often fail in real-world applications.

In response, Molodtsov [2] introduced soft set theory, a parameter-based paradigm designed to overcome these limitations. Maji et al. [3] introduced basic operations, later reinterpreted by Pei et al. [4] from an information-theoretic perspective, which significantly advanced the theory. Ali et al. [5] enhanced the operational flexibility of the theory by introducing restricted and extended operations. Earlier contributions [6–19] had clarified ambiguities, expanded the algebra of soft operations, and generalized soft equalities.

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In recent years, the systematic introduction and formal algebraic analysis of newly defined operations have significantly strengthened this foundation. The combined efforts of [20–34] have resulted in a powerful, extensible, and internally consistent algebraic framework that supports the further theoretical development of soft set theory. Parallel research focused on soft subsethood and equality. Pei et al. [4], Feng et al. [35], and Qin and Hong [36] first generalized these concepts, while Jun and Yang [37] and Liu et al. [38] extended them further through J-soft and L-soft equalities. Feng et al. [39] categorized soft subsets under L-equality, showing that specific quotient structures yield semigroup properties.

Broader generalizations, including g-soft, gf-soft, and T-soft equalities, embedded congruence and lattice-theoretic mechanisms into soft algebra [40–43]. Çağman and Enginoğlu [44] established a coherent axiomatic basis for soft sets through a crucial reformulation. Building on this foundation, binary operations have been defined over classical algebraic structures. The soft union–intersection product has been studied in group-theoretic [45], semigroup-theoretic [46], and ring-theoretic [47] contexts, while soft intersection–union products have been extended to rings [48], semigroups [49], and groups [50].

This study builds on this rich foundation by introducing a novel binary operation on soft sets indexed by group-structured parameter domains, namely the soft intersection–gamma product. We investigate whether the operation satisfies closure, associativity, commutativity, idempotency, and distributivity. The operation is defined within a formally consistent axiomatic system. We conduct a thorough analysis of its behavior with respect to identity, absorbing, null, and absolute soft sets. Importantly, the operation is compatible with equality and generalized soft subsethood, allowing it to be incorporated into existing soft algebraic frameworks. Its expressive power and structural coherence within soft subset hierarchies are demonstrated through comparisons with earlier soft operations. By simulating traditional group-theoretic behavior in a soft setting, this work expands the algebraic foundations of soft set theory and provides a basis for developing generalized soft group theory. Additionally, the operation supports applications in algebraic classification, abstract algebra, and uncertainty-aware computation, thereby enhancing both the theoretical depth and practical usefulness of soft set frameworks. The remainder of this manuscript is organized as follows. Section 2 presents important definitions and formal preliminaries. Section 3 introduces the soft intersection–gamma product and develops its algebraic theory in detail. Section 4 summarizes the main theoretical findings and outlines potential directions for strengthening the algebraic foundations of soft sets and exploring their applications in uncertainty modeling and abstract algebra.

2. Preliminaries

Soft set theory, a parameter-dependent framework for representing epistemic uncertainty, was first introduced by Molodtsov [2]. However, the original formulation lacked the algebraic rigor required for formal development. This limitation was addressed by Çağman and Enginoğlu [44], whose axiomatic refinement resolved structural irregularities and established a logical, algebraically sound foundation. This refined framework underpins the present study and serves as the reference point for all subsequent definitions, operations, and algebraic constructions. Unless otherwise stated, all subsequent references to soft sets and their operations should be understood within this refined formalism.

Definition 2.1. [44] Let E be a parameter set, U be a universal set, $P(U)$ be the power set of U, and $\mathcal{H} \subseteq E$. Then, the soft set $f_{\mathcal{H}}$ over U is a function such that $f_{\mathcal{H}}: E \to P(U)$, where for all $w \notin \mathcal{H}$, $f_{\mathcal{H}}(w) = \emptyset$. That is,

$$f_{\mathcal{H}} = \{(w, f_{\mathcal{H}}(w)) : w \in E\}$$

From now on, the soft set over U is abbreviated by $\mathcal{SS}$.

Definition 2.2. [44] Let $f_{\mathcal{H}}$ be an $\mathcal{SS}$. If $f_{\mathcal{H}}(w) = \emptyset$ for all $w \in E$, then $f_{\mathcal{H}}$ is called a null SS and denoted by $\emptyset_E$, and if $f_{\mathcal{H}}(w) = U$, for all $w \in E$, then $f_{\mathcal{H}}$ is called an absolute SS and indicated by U_E.

Definition 2.3. [44] Let $f_{\mathcal{H}}$ and $g_{\aleph}$ be two $\mathcal{SS}$s. If $f_{\mathcal{H}}(w) \subseteq g_{\aleph}(w)$, for all $w \in E$, then $f_{\mathcal{H}}$ is a soft subset of $g_{\aleph}$ and denoted by $f_{\mathcal{H}} \tilde{\subseteq} g_{\aleph}$. If $f_{\mathcal{H}}(w) = g_{\aleph}(w)$, for all $w \in E$, then $f_{\mathcal{H}}$ is called soft equal to $g_{\aleph}$, and denoted by $f_{\mathcal{H}} = g_{\aleph}$.

Definition 2.4. ([44] Let $f_{\mathcal{H}}$ and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, the intersection of $f_{\mathcal{H}}$ and $g_{\aleph}$ is the $\mathcal{SS}$ $f_{\mathcal{H}} \tilde{\cap} g_{\aleph}$, where $(f_{\mathcal{H}} \tilde{\cap} g_{\aleph})(w) = f_{\mathcal{H}}(w) \cap g_{\aleph}(w)$, for all $w \in E$.

Definition 2.5. [44] Let $f_{\mathcal{H}}$ be an $\mathcal{SS}$. Then, the complement of $f_{\mathcal{H}}$ denoted by $f_{\mathcal{H}}{}^{c}$, is defined by the soft set $f_{\mathcal{H}}{}^{c}: E \to P(U)$ such that $f_{\mathcal{H}}{}^{c}(e) = U \backslash f_{\mathcal{H}}(e) = (f_{\mathcal{H}}(e))'$, for all $e \in E$.

Definition 2.6. [51] Let f_K and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, f_K is called a soft S-subset of $g_{\aleph}$, denoted by $f_K \tilde{\subseteq}_S g_{\aleph}$ if for all $w \in E$, $f_K(w) = \mathcal{M}$ and $g_{\aleph}(w) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} \subseteq \mathcal{D}$. Moreover, two SSs f_K and $g_{\aleph}$ are said to be soft S-equal, denoted by $f_K =_S g_{\aleph}$, if $f_K \tilde{\subseteq}_S g_{\aleph}$ and $g_{\aleph} \tilde{\subseteq}_S f_K$.

It is obvious that if $f_K =_S g_{\aleph}$, then f_K and $g_{\aleph}$ are the same constant functions, that is, for all $w \in E$, $f_K(w) = g_{\aleph}(w) = \mathcal{M}$, where $\mathcal{M}$ is a fixed set .

Definition 2.7. [51] Let f_K and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, f_K is called a soft A-subset of $g_{\aleph}$, denoted by $f_K \tilde{\subseteq}_A g_{\aleph}$, if, for each $a, b \in E$, $f_K(a) \subseteq g_{\aleph}(b)$.

Definition 2.8. [51] Let f_K and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, f_K is called a soft S-complement of $g_{\aleph}$, denoted by $f_K =_S (g_{\aleph})^c$, if, for all $w \in E$, $f_K(w) = \mathcal{M}$ and $g_{\aleph}(w) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} = \mathcal{D}'$. Here, $\mathcal{D}' = U \backslash \mathcal{D}$.

From now on, let G be a group, and $S_G(U)$ denotes the collection of all $\mathcal{SS}$s over U, whose parameter sets are G; that is, each element of $S_G(U)$ is an $\mathcal{SS}$ parameterized by G.

Definition 2.9. [51] Let f_G and g_G be two $\mathcal{SS}$s. Then, the soft union-lambda product $f_G \otimes_{i/d} g_G$ is defined by

$$\begin{aligned}(f_G \otimes_{u/l})(x) &= \bigcup_{x=yz}\left(f_G(y)\lambda g_G(z)\right)\\ &= \bigcup_{x=yz}\left(f_G(z) \cup \left(g_G(y)\right)'\right)\\ &= \bigcup_{x=yz}\left(f_G(y) \cap \left(g_G\right)^c(z)\right),\\ & y,z \in G\end{aligned}$$

for all $x \in G$.

For additional information on SSs, we refer to [52–77].

3. Soft Intersection-Gamma Product of Groups

The soft intersection-gamma product is a new binary operation on soft sets that is introduced and formally defined in this section. It is built over parameter domains with group-theoretic structure. Its basic structural features (closure, associativity, commutativity, and idempotency) and compatibility with expanded ideas of soft equality and subsethood are established by an extensive algebraic analysis. The behavior of the operation in soft inclusion hierarchies and its consistency with the larger algebraic terrain of soft set theory are given special attention.

Definition 3.1. Let f_G and g_G be two $\mathcal{SS}$s. Then, the soft intersection-gamma product $f_G \otimes_{i/g} g_G$ is defined by

$$\begin{aligned}(f_G \otimes_{i/g} g_G)(x) &= \bigcap_{x=yz}\left(f_G(y)\gamma g_G(z)\right)\\ &= \bigcap_{x=yz}\left(f_G{}^c(y) \cap g_G(z)\right)\\ &= \bigcap_{x=yz}\left(\left(f_G(y)\right)' \cap g_G(z)\right),\\ & y,z \in G\end{aligned}$$

for all $x \in G$.

Note here that $A\gamma B = A' \cap B$, where A and B are fixed sets. For more on gamma (γ) operation of sets, we refer to [78]. It is evident that since G is a group, there always exist $y,z \in G$ such that $x = yz$, for all $x \in G$.Let the order of the group G be n, that is, $|G| = n$. Then, it is obvious that there exist n distinct representations for each $x \in G$ such that $x = yz$, where $y,z \in G$.

In [79–81], soft lambda, soft star and soft gamma product are proposed, respectively. However, there is a big difference between these products and the soft intersection-gamma product proposed in this paper. Let $f_G,\ g_G \in S_G(U)$. Then, the parameter set of f_G soft lambda (or soft star and soft gamma product) g_G is GxG, whereas the parameter set of f_G soft intersection-gamma product g_G is G.

Note 3.2. The soft intersection-gamma product is well-defined in $S_G(U)$. In fact, let $f_G,\ g_G,\ o_G,\ k_G \in S_G(U)$ such that $(f_G, g_G) = (o_G, k_G)$. Then, $f_G = o_G$ and $g_G = k_G$, implying that $f_G(x) = o_G(x)$ and $g_G(x) = k_G(x)$ for all $x \in G$. Thereby,

$$\begin{aligned}(f_G \otimes_{i/g} g_G)(x) &= \bigcap_{x=yz}\left(f_G{}^c(y) \cap g_G(z)\right)\\ &= \bigcap_{x=yz}\left(o_G{}^c(y) \cap k_G(z)\right)\\ &= \left(o_G \otimes_{i/g} k_G\right)(x)\end{aligned}$$

Hence, $f_G \otimes_{i/g} g_G = o_G \otimes_{i/g} k_G$.

Example 3.3. Consider the group $\mathrm{G} = \{\text{Ձ}, \text{Ϭ}\}$ with the following operation**:**

$\cdot$	Ձ	Ϭ
Ձ	Ձ	Ϭ
Ϭ	Ϭ	Ձ

Let f_G and g_G be two $\mathcal{SS}$s over $U = D_2 = \{< x,y >: x^2 = y^2 = e, xy = yx\} = \{e, x, y, yx\}$ as follows:

$f_G = \{(\text{Ձ}, \{e,x,y\}), (\text{Ϭ}, \{e,yx\})\}$ and $g_G = \{(\text{Ձ}, \{x,yx\}), (\text{Ϭ}, \{e,y\})\}$

Since $\text{Ձ} = \text{ՁՁ} = \text{ϬϬ}$, $(f_G \otimes_{i/g} g_G)(\text{Ձ}) = \left(f_G{}^c(\text{Ձ}) \cap g_G(\text{Ձ})\right) \cap \left(f_G{}^c(\text{Ϭ}) \cap g_G(\text{Ϭ})\right) = \emptyset$ and since $\text{Ϭ} = \text{ՁϬ} = \text{ϬՁ}$, $(f_G \otimes_{i/g} g_G)(\text{Ϭ}) = \left(f_G{}^c(\text{Ձ}) \cap g_G(\text{Ϭ})\right) \cap \left(f_G{}^c(\text{Ϭ}) \cap g_G(\text{Ձ})\right) = \emptyset$ is obtained. Hence,

$$f_G \otimes_{i/g} g_G = \{(\text{Ձ}, \emptyset), (\text{Ϭ}, \emptyset)\}$$

Proposition 3.4. The set $S_G(U)$ is closed under the soft intersection-gamma product. That is, if f_G and g_G are two $\mathcal{SS}$s, then so is $f_G \otimes_{i/g} g_G$.

PROOF. It is obvious that the soft intersection-gamma product is a binary operation in $S_G(U)$. Thereby, $S_G(U)$ is closed under the soft intersection-gamma product.

Proposition 3.5. The soft intersection-gamma product is not associative in $S_G(U)$

PROOF. Consider the group G and the $\mathcal{SS}$s f_G and g_G in Example 3.3. Let $\hbar_G$ be an $\mathcal{SS}$s over $U = \{e, x, y, yx\}$ such that $\hbar_G = \{(\mathfrak{a}, \{x, y\}), (\mathfrak{b}, \{x\})\}$.

Since $f_G \otimes_{i/g} g_G = \{(\mathfrak{a}, \emptyset), (\mathfrak{b}, \emptyset)\}$, then

$$(f_G \otimes_{i/g} g_G) \otimes_{i/g} \hbar_G = \{(\mathfrak{a}, \{x\}), (\mathfrak{b}, \{x\})\}$$

Moreover, since $g_G \otimes_{i/g} \hbar_G = \{(\mathfrak{a}, \emptyset), (\mathfrak{b}, \emptyset)\}$, then

$$f_G \otimes_{i/g} (g_G \otimes_{i/g} \hbar_G) = \{(\mathfrak{a}, \emptyset), (\mathfrak{b}, \emptyset)\}$$

Thereby, $(f_G \otimes_{i/g} g_G) \otimes_{i/g} \hbar_G \neq f_G \otimes_{i/g} (g_G \otimes_{i/g} \hbar_G)$.

Proposition 3.6. The soft intersection-gamma product is not commutative in $S_G(U)$.

PROOF. Consider the group G in Example 3.3. Let f_G and g_G be two $\mathcal{SS}$s over $U = \{e, x, y, yx\}$ as follows:

$$f_G = \{(\mathfrak{a}, \{e, x\}), (\mathfrak{b}, \{e\})\} \text{ and } g_G = \{(\mathfrak{a}, \{e, x, yx\}), (\mathfrak{b}, \{e, yx\})\}$$

$f_G \otimes_{i/g} g_G = \{(\mathfrak{a}, \{yx\}), (\mathfrak{b}, \{yx\})\}$, and $g_G \otimes_{i/g} f_G = \{(\mathfrak{a}, \emptyset), (\mathfrak{b}, \emptyset)\}$

implying that $f_G \otimes_{i/g} g_G \neq g_G \otimes_{i/g} f_G$.

Proposition 3.7. The soft intersection-gamma product is not idempotent in $S_G(U)$.

PROOF. Consider the f_G $\mathcal{SS}$ in Example 3.3. Then, for all $x \in G$,

$$f_G \otimes_{u/g} f_G = \{(\mathfrak{a}, \emptyset), (\mathfrak{b}, \{x, y, yx\})\}$$

implying that $f_G \otimes_{i/g} f_G \neq f_G$.

Proposition 3.8. Let f_G be a constant $\mathcal{SS}$. Then, $f_G \otimes_{i/g} f_G = \emptyset_G$.

PROOF. Let f_G be a constant $\mathcal{SS}$ such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$(f_G \otimes_{u/g} f_G)(x) = \bigcap_{x=yz} \left(f_G{}^c(y) \cap f_G(z)\right) = \emptyset_G(x)$$

Thereby, $f_G \otimes_{i/g} f_G = \emptyset_G$.□

Remark 3.9. Let $S_G{}^*(U)$ be the collection of all constant $\mathcal{SS}$s. Then, the soft intersection-gamma product is not idempotent in $S_G{}^*(U)$ either.

Proposition 3.10. $\emptyset_G$ is the right absorbing element of the soft intersection-gamma product in $S_G(U)$.

PROOF. Let f_G be an $\mathcal{SS}$. Then, for all $x \in G$,

$$\begin{aligned}(f_G \otimes_{i/g} \emptyset_G)(x) &= \bigcap_{x=yz} \left(f_G{}^c(y) \cap \emptyset_G(z)\right) \\ &= \bigcap_{x=yz} \left(f_G{}^c(y) \cap \emptyset\right) \\ &= \emptyset_G(x)\end{aligned}$$

Thus, $f_G \otimes_{i/g} \emptyset_G = \emptyset_G$.

Proposition 3.11. $\emptyset_G$ is not the left absorbing element of the soft intersection-gamma product in $S_G(U)$.

PROOF. Consider the $\mathcal{SS}$ f_G in Example 3.3. Then,

$$(\emptyset_G \otimes_{i/g} f_G)(x) = \{(a, \{e\}), (b, \{e\})\}$$

implying that $\emptyset_G \otimes_{i/g} f_G \neq \emptyset_G$.

Remark 3.12. $\emptyset_G$ is not the absorbing element of the soft intersection-gamma product in $S_G(U)$.

Proposition 3.13. Let f_G be a constant $\mathcal{SS}$. Then, $\emptyset_G \otimes_{i/g} f_G = f_G$.

PROOF**.** Let f_G be a constant $\mathcal{SS}$ such that, for all $x \in G$, $f_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\begin{aligned}(\emptyset_G \otimes_{i/g} f_G)(x) &= \bigcap_{x=yz} \left(\emptyset_G{}^c(y) \cap f_G(z)\right) \\ &= \bigcap_{x=yz} \left(U_G(y) \cap f_G(z)\right) = f_G(x)\end{aligned}$$

Thereby, $\emptyset_G \otimes_{i/g} f_G = f_G$.□

Remark 3.14., $\emptyset_G$ is the left identity element and the absorbing element of the soft intersection-gamma product in $S_G{}^*(U)$.

Proposition 3.15. Let f_G be an $\mathcal{SS}$. Then, $U_G \otimes_{i/g} f_G = \emptyset_G$.

PROOF**.** Let f_G be an $\mathcal{SS}$. Then, for all $x \in G$,

$$\begin{aligned}(U_G \otimes_{i/g} f_G)(x) &= \bigcap_{x=yz} \left(U_G{}^c(y) \cap f_G(z)\right) \\ &= \bigcap_{x=yz} \left(\emptyset \cap f_G(z)\right) = \emptyset_G(x)\end{aligned}$$

Thereby, $U_G \otimes_{i/g} f_G = \emptyset_G$.□

Proposition 3.16. Let f_G be a constant $\mathcal{SS}$. Then, $f_G \otimes_{i/g} U_G = f_G{}^c$.

PROOF. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathscr{f}_G\,(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\left(\mathscr{f}_G \otimes_{i/g} U_G\right)(x) = \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cap U_G(z)\right)$$
$$= \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cap U\right) = \mathscr{f}_G{}^c(x)$$

Thereby, $\mathscr{f}_G \otimes_{i/g} U_G = \mathscr{f}_G{}^c$.

Proposition 3.17. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$. Then, $\mathscr{f}_G{}^c \otimes_{i/g} \mathscr{f}_G = \mathscr{f}_G$.

PROOF. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathscr{f}_G\,(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\left(\mathscr{f}_G{}^c \otimes_{i/g} \mathscr{f}_G\right)(x) = \bigcap_{x=yz} \left((\mathscr{f}_G{}^c)^c(y) \cap \mathscr{f}_G(z)\right)$$
$$= \bigcap_{x=yz} \left(\mathscr{f}_G(y) \cap \mathscr{f}_G(z)\right) = \mathscr{f}_G(x)$$

Thereby, $\mathscr{f}_G{}^c \otimes_{i/g} \mathscr{f}_G = \mathscr{f}_G$.□

Proposition 3.18. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$. Then, $\mathscr{f}_G \otimes_{i/g} \mathscr{f}_G{}^c = \mathscr{f}_G{}^c$.

PROOF. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathscr{f}_G\,(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\left(\mathscr{f}_G \otimes_{i/g} \mathscr{f}_G{}^c\right)(x) = \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cap \mathscr{f}_G{}^c(z)\right) = \mathscr{f}_G{}^c(x)$$

Thereby, $\mathscr{f}_G \otimes_{i/g} \mathscr{f}_G{}^c = \mathscr{f}_G{}^c$.□

Theorem 3.19. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s. Then, $\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G = U_G$ if and only if $\mathscr{f}_G = \emptyset_G$ and $\mathscr{g}_G = U_G$.

PROOF. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s. Suppose that $\mathscr{f}_G = \emptyset_G$ and $\mathscr{g}_G = U_G$. Then, $\mathscr{f}_G(x) = \emptyset_G(x) = \emptyset$ and $\mathscr{g}_G(x) = U_G(x) = U$, for each $x \in G$. Thus, for all $x \in G$,

$$\left(\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G\right)(x) = \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cap \mathscr{g}_G(z)\right)$$
$$= \bigcap_{x=yz} \left(\emptyset_G{}^c(y) \cap U_G(z)\right)$$
$$= \bigcap_{x=yz} \left(U_G(y) \cap U_G(z)\right)$$
$$= U_G(x)$$

Thereby, $\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G = U_G$.

Conversely, suppose that $\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G = U_G$. Then, $\left(\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G\right)(x) = U_G(x) = U$, for all $x \in G$. Thus, for all $x \in G$,

$$U_G(x) = U = \left(\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G\right)(x) = \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cap \mathscr{g}_G(z)\right)$$

This,implies that $\mathscr{f}_G{}^c(y) \cap \mathscr{g}_G(z) = U$, for all $y, z \in G$. Thus, $\mathscr{f}_G(x) = \emptyset_G(x) = \emptyset$ and $\mathscr{g}_G(x) = U_G(x) = U$ for each $x \in G$. Thereby, $\mathscr{f}_G = \emptyset_G$ and $\mathscr{g}_G = U_G$.

Proposition 3.20. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s. If one of the following assertions is satisfied, then $\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G = \emptyset_G$:

i. $\mathscr{g}_G = \emptyset_G$
ii. $\mathscr{f}_G = U_G$
iii. $\mathscr{g}_G \subseteq_A \mathscr{f}_G$

PROOF. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s over U.

i. It follows from Proposition 3.10.

ii. It follows from Proposition 3.15

iii. Let $\mathscr{g}_G \subseteq_A \mathscr{f}_G$. Then, for each $x, y \in G$, $\mathscr{g}_G(x) \subseteq \mathscr{f}_G(y)$. Thus, for all $x \in G$,

$$\left(\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G\right)(x) = \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cap \mathscr{g}_G(z)\right) = \emptyset_G(x) = \emptyset$$

Thereby, $\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G = \emptyset_G$.

Proposition 3.21. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s. If $\mathscr{f}_G{}^c \,\tilde{\subseteq}_S\, \mathscr{g}_G$, then $\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G = \mathscr{f}_G{}^c$.

PROOF. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s and $\mathscr{f}_G{}^c \,\tilde{\subseteq}_S\, \mathscr{g}_G$. Hence, for all $x \in G$, $\mathscr{f}_G(x) = A$ and $\mathscr{g}_G(x) = B$, where A and B are two fixed sets and $A' \subseteq B$. Thus, for all $x \in G$,

$$\left(\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G\right)(x) = \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cap \mathscr{g}_G(z)\right) = \mathscr{f}_G{}^c(x)$$

Thereby, $\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G = \mathscr{f}_G{}^c$. □

Proposition 3.22. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s. If $\mathscr{g}_G \,\tilde{\subseteq}_S\, (\mathscr{f}_G)^c$ and $\mathscr{g}_G$ be a constant $\mathcal{SS}$, then $\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G = \mathscr{g}_G$.

PROOF. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s and $\mathscr{g}_G \,\tilde{\subseteq}_S\, (\mathscr{f}_G)^c$ and for all $x \in G$, $\mathscr{g}_G\,(x) = A$, where A is a fixed set. Hence, for all $x \in G$, $\mathscr{f}_G(x) = A$ and $\mathscr{g}_G(x) = B$, where A and B are two fixed sets and $B \subseteq A'$. Thus, for all $x \in G$,

$$\left(\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G\right)(x) = \bigcap_{x=yz} \left(\mathscr{f}_G{}^c(y) \cap \mathscr{g}_G(z)\right) = \mathscr{g}_G(x)$$

Thereby, $\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G = \mathscr{g}_G$. □

Proposition 3.23. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s. Then, $\left(\mathscr{f}_G \otimes_{i/g} \mathscr{g}_G\right)^{\mathrm{c}} = \mathscr{f}_G \otimes_{i/l} \mathscr{g}_G$.

PROOF. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s. Then, for all $x \in G$,

$$\begin{aligned}(f_G \otimes_{i/g} g_G)^{c}(x) &= \left(\bigcap_{x=yz} \left(f_G{}^{c}(y) \cap g_G(z)\right)\right)' \\ &= \bigcup_{x=yz} \left(f_G{}^{c}(y) \cap g_G(z)\right)' \\ &= \bigcup_{x=yz} \left(f_G(y) \cup g_G{}^{c}(z)\right) \\ &= (f_G \otimes_{u/l} g_G)(x)\end{aligned}$$

Thereby, $(f_G \otimes_{i/g} g_G)^{c} = f_G \otimes_{u/l} g_G$.

Proposition 3.24. Let f_G, g_G, and h_G be three SSs. If $f_G \tilde{\subseteq} g_G$, then $g_G \otimes_{i/g} h_G \tilde{\subseteq} f_G \otimes_{i/g} h_G$ and $h_G \otimes_{i/g} f_G \tilde{\subseteq} h_G \otimes_{i/g} g_G$.

PROOF. Let f_G, g_G, and h_G be three SSs such that $f_G \tilde{\subseteq} g_G$. Then, for all $x \in G$, $f_G(x) \subseteq g_G(x)$ and $g_G{}^{c}(x) \subseteq f_G{}^{c}(x)$. Thus, for all $x \in G$,

$$\begin{aligned}(g_G \otimes_{i/g} h_G)(x) &= \bigcap_{x=yz} \left(g_G{}^{c}(y) \cap h_G(z)\right) \\ &\subseteq \bigcap_{x=yz} \left(f_G{}^{c}(y) \cap h_G(z)\right) \\ &= (f_G \otimes_{i/g} h_G)(x)\end{aligned}$$

implying that $g_G \otimes_{i/g} h_G \tilde{\subseteq} f_G \otimes_{i/g} h_G$. Similarly, for all $x \in G$,

$$\begin{aligned}(h_G \otimes_{i/g} f_G)(x) &= \bigcap_{x=yz} \left(h_G{}^{c}(y) \cap f_G(z)\right) \\ &\subseteq \bigcap_{x=yz} \left(h_G{}^{c}(y) \cap g_G(z)\right) \\ &= (h_G \otimes_{i/g} g_G)(x)\end{aligned}$$

implying that $h_G \otimes_{i/g} f_G \tilde{\subseteq} h_G \otimes_{i/g} g_G$.□

Proposition 3.25. Let f_G, g_G, σ_G, and k_G be four SSs. If $k_G \tilde{\subseteq} \sigma_G$, and $f_G \tilde{\subseteq} g_G$, then $\sigma_G \otimes_{i/g} f_G \tilde{\subseteq} k_G \otimes_{i/g} g_G$ and $g_G \otimes_{i/g} k_G \tilde{\subseteq} f_G \otimes_{i/g} \sigma_G$.

PROOF. Let f_G, g_G, σ_G, and k_G be four SSs such that $k_G \tilde{\subseteq} \sigma_G$, and $f_G \tilde{\subseteq} g_G$. Then, for all $x \in G$, $k_G(x) \subseteq \sigma_G(x)$ and $f_G(x) \subseteq g_G(x)$, and thus, $\sigma_G{}^{c}(x) \subseteq k_G{}^{c}(x)$, $g_G{}^{c}(x) \subseteq f_G{}^{c}(x)$. Then, for all $x \in G$,

$$\begin{aligned}(\sigma_G \otimes_{i/g} f_G)(x) &= \bigcap_{x=yz} \left(\sigma_G{}^{c}(y) \cap f_G(z)\right) \\ &\subseteq \bigcap_{x=yz} \left(k_G{}^{c}(y) \cap g_G(z)\right) \\ &= (k_G \otimes_{i/g} g_G)(x)\end{aligned}$$

implying that $\sigma_G \otimes_{i/g} f_G \tilde{\subseteq} k_G \otimes_{i/g} g_G$. Similarly, for all $x \in G$,

$$\begin{aligned}(g_G \otimes_{i/g} k_G)(x) &= \bigcap_{x=yz} \left(g_G{}^{c}(y) \cap k_G(z)\right) \\ &\subseteq \bigcap_{x=yz} \left(f_G{}^{c}(y) \cap \sigma_G(z)\right) \\ &= (f_G \otimes_{i/g} \sigma_G)(x)\end{aligned}$$

is obtained implying that $g_G \otimes_{i/g} k_G \tilde{\subseteq} f_G \otimes_{i/g} \sigma_G$. □

Proposition 3.26. The soft intersection-gamma product distributes over the intersection operation of SSs from the left side.

PROOF. Let f_G, g_G, and h_G be three SSs. Then, for all $x \in G$,

$$\begin{aligned}\left(f_G \otimes_{i/g} (g_G \tilde{\cap} h_G)\right)(x) &= \bigcap_{x=yz} \left(f_G{}^{c}(y) \cap (g_G \tilde{\cap} h_G)(z)\right) \\ &= \bigcap_{x=yz} \left(f_G{}^{c}(y) \cap \left(g_G(z) \cap h_G(z)\right)\right) \\ &= \bigcap_{x=yz} \left(\left(f_G{}^{c}(y) \cap g_G(z)\right) \cap \left(f_G{}^{c}(y) \cap h_G(z)\right)\right) \\ &= \left[\bigcap_{x=yz} \left(f_G{}^{c}(y) \cap g_G(z)\right)\right] \cap \left[\bigcap_{x=yz} \left(f_G{}^{c}(y) \cap h_G(z)\right)\right] \\ &= (f_G \otimes_{i/g} g_G)(x) \cap (f_G \otimes_{i/g} h_G)(x)\end{aligned}$$

Thus, $f_G \otimes_{i/g} (g_G \tilde{\cap} h_G) = (f_G \otimes_{i/g} g_G) \tilde{\cap} (f_G \otimes_{i/g} h_G)$.□

Example 3.27. Consider the group G in Example 3.3. Let f_G, g_G, and h_G be three SSs over $U = \{e, x, y, yx\}$ as follows:

$f_G = \{(\mathfrak{Q}, \{e, x, y\}), (\mathfrak{b}, \{e, yx\})\}$, $g_G = \{(\mathfrak{Q}, \{x, yx\}), (\mathfrak{b}, \{e, y\})\}$, and $h_G = \{(\mathfrak{Q}, \{x, y\}), (\mathfrak{b}, \{x\})\}$

Since $f_G \otimes_{i/g} g_G = \{(\mathfrak{Q}, \emptyset), (\mathfrak{b}, \emptyset)\}$ and $f_G \otimes_{i/g} h_G = \{(\mathfrak{Q}, \emptyset), (\mathfrak{b}, \emptyset)\}$, then

$$(f_G \otimes_{i/g} g_G) \tilde{\cap} (f_G \otimes_{i/g} h_G) = \{(\mathfrak{Q}, \emptyset), (\mathfrak{b}, \emptyset)\}$$

Moreover, since $g_G \tilde{\cap} h_G = \{(\mathfrak{Q}, \{x\}), (\mathfrak{b}, \emptyset)\}$, then

$$f_G \otimes_{i/g} (g_G \tilde{\cap} h_G) = \{(\mathfrak{Q}, \emptyset), (\mathfrak{b}, \emptyset)\}$$

Thus, $f_G \otimes_{i/g} (g_G \tilde{\cap} h_G) = (f_G \otimes_{i/g} g_G) \tilde{\cap} (f_G \otimes_{i/g} h_G)$.□

Proposition 3.28. The soft intersection-gamma product does not distribute over the intersection operation of $\mathcal{SS}$s from right side.

PROOF. Consider the group G in Example 3.3. Let $\mathscr{f}_G$, $\mathscr{g}_G$, and $\hbar_G$ be three $\mathcal{SS}$s over $U=\{e,x,y,yx\}$ as follows:

$$\mathscr{f}_G=\{(\mathfrak{Q},\{e,x,y\}),(\mathfrak{b},\{e,yx\})\},\ \mathscr{g}_G=\{(\mathfrak{Q},\{x,yx\}),(\mathfrak{b},\{e,y\})\},\text{ and } \hbar_G=\{(\mathfrak{Q},\{x,yx\}),(\mathfrak{b},\{y,yx\})\}$$

Since $\mathscr{f}_G\otimes_{i/g}\hbar_G=\{(\mathfrak{Q},\emptyset),(\mathfrak{b},\emptyset)\}$ and $\mathscr{g}_G\otimes_{i/g}\hbar_G=\{(\mathfrak{Q},\emptyset),(\mathfrak{b},\emptyset)\}$, then

$$\left(\mathscr{f}_G\otimes_{i/g}\hbar_G\right)\tilde{\cap}\left(\mathscr{g}_G\otimes_{i/g}\hbar_G\right)=\{(\mathfrak{Q},\emptyset),(\mathfrak{b},\emptyset)\}$$

Moreover, since $\mathscr{f}_G\ \tilde{\cap}\ \mathscr{g}_G=\{(\mathfrak{Q},\{x\}),(\mathfrak{b},\{e\})\}$, then

$$(\mathscr{f}_G\ \tilde{\cap}\ \mathscr{g}_G)\otimes_{i/g}\hbar_G=\{(\mathfrak{Q},\{yx\}),(\mathfrak{b},\{yx\})\}$$

Thus, $(\mathscr{f}_G\ \tilde{\cap}\ \mathscr{g}_G)\otimes_{i/g}\hbar_G\neq\left(\mathscr{f}_G\otimes_{i/g}\hbar_G\right)\tilde{\cap}\left(\mathscr{g}_G\otimes_{i/g}\hbar_G\right)$.□

Remark 3.29. The soft intersection-gamma product does not distribute over the intersection operation of $\mathcal{SS}$s from both sides.

4. Conclusion

This study introduces a new binary operation on soft sets, called the soft intersection–gamma product, defined over parameter domains with group-theoretic structures. The operation is examined through a detailed algebraic analysis, focusing on its behavior with respect to soft subset hierarchies and its compatibility with generalized soft equality. It is systematically compared with other established binary soft operations to highlight differences in expressive capacity and structural consistency. The interaction of the operation with null and absolute soft sets, as well as with other group-based soft products, is also analyzed to clarify its role within the algebraic framework of soft set theory. The analysis is developed under a rigorous axiomatic setting and considers fundamental algebraic properties such as closure, associativity, commutativity, idempotency, distributivity, and the presence or absence of identity, inverse, and absorbing elements. The findings confirm the coherence and formal soundness of the soft intersection–gamma product, establishing it as a foundational extension of classical algebraic structures into soft set theory. More broadly, the operation provides a conceptual basis for the advancement of generalized soft group theory, where soft sets indexed by group-structured parameters reflect group-like behavior through rigorously defined operations. The framework also supports future research on algebraic development and practical applications in abstract modeling, generalized soft equalities, and uncertainty-aware decision-making.

Author contributions

ZA: Investigation, Visualization, Conceptualization, Writing-Review, Validation.
AS: Supervision, Visualization, Conceptualization, Validation, Review.

Conflict of Interest

The authors have no conflicts of interest to declare.

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