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RESEARCH ARTICLE

Optimization of Two-Dimensional Acoustic Diffuser Surfaces Using Kirchhoff Approximation in MATLAB

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ABSTRACT

This study numerically analyzes the angular reflection performance of two-dimensional acoustic diffuser surfaces using the Kirchhoff approach in MATLAB. In the study, ideal Lambertian and band-limited diffuser types are compared under different incidence angles and surface geometries (slope and recess width). The aim is to reveal the effect of surface design parameters on the angular distribution of sound energy and to contribute to the design of structures that provide homogeneous reflection. The results show that Lambertian surfaces provide ideal reflection only at normal incidence angles; band-limited diffusers can direct the energy in a controlled manner within a certain angle range. It is observed that the reflection intensity decreases and the distribution widens with the increase in slope. This situation reveals the decisive role of surface geometry on acoustic performance. The findings provide an important basis for surface optimization in acoustic design applications.

Contents

1. Introduction

Acoustic design considers spatial and physical factors in order to optimize the direction, propagation and perception of sound. Effective control of sound in closed spaces, reducing reverberation time, balancing reflection density and increasing communication clarity are among key aims in acoustic design. To serve for this purpose, acoustic diffuser surfaces are utilized to spread the energy homogeneously throughout the environment by dispersing sound waves in different directions.

Various approaches have been developed to model the reflection and scattering behavior of sound upon interaction with surfaces that have distinctive characteristics and diverse effects on the echo time and sound intensity [1, 2]. The change in surface geometry and roughness directly determines the acoustic comfort by affecting the angles at which the sound is concentrated or dispersed. Diffuser types such as QRD (Quadratic Residue Diffuser) developed in this context perform at certain frequencies, but cannot provide sufficient distribution in wide frequency ranges and deviate from angular homogeneity [3, 4].

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However, technologies that model the interaction of sound waves with different surfaces are becoming widespread today and acoustic performance can be evaluated in advance through digital simulations [5]. However, the success of these simulations depends on the extent to which the model used represents physical reality. Existing studies are generally limited to limited geometry types and incidence angles; in addition, the effects of the physical dimensions of the diffusers and surface slopes on performance are not detailed enough [6, 7].

Studies in the field of acoustic design have focused on the development of different methods and technologies to improve the distribution of sound within the environment. The basis of these studies is formed by phenomena such as reflection, scattering and echo, which affect the spatial behavior of sound. In particular, it is emphasized in many studies that physical parameters such as surface roughness, material structure and diffuser geometry directly affect acoustic energy distribution [1, 6].

Surface roughness and geometry change the way sound propagates. Bourlier [8] and Pignier et al. [2] examined the relationship between reflection and surface structure, demonstrating how echo time is critical to communication clarity. Dehghani et al. [9] the automatic shape optimization of diffusers and uses CFD (Computational Fluid Dynamics) simulations and surrogate modeling methods in this process. Similarly, Shi et al. [7] showed that irregularities and shape factors on surfaces can affect acoustic homogeneity by causing sound to scatter with deviations from the direction of incidence.

In more technical studies on acoustic diffusers, QRD (Quadratic Residue Diffuser) designs based on number theory are particularly noteworthy. Kleiner et al. [4] experimentally investigated the reflection and scattering properties of diffusers with different surface structures using scaled models; Döşemeciler [3] proposed optimizing two-dimensional diffuser designs in the context of architectural acoustics by modeling them with mathematical approaches. However, these studies did not comprehensively address the effects of geometric parameters such as surface slope and width at different incidence angles.

Cushing et al. [5] and Humeida et al. [10] emphasized the importance of technological tools in the evaluation of acoustic performance and discussed the use of ultrasonic imaging and acoustic simulation software. Martin [11] developed a hybrid model to simulate the diffusion of first reflections in two-dimensional acoustic environments. However, most of these approaches either work in limited frequency ranges or cannot be widely used in practical applications because they require complex modeling.

In this context, the Kirchhoff approach has found a place in the literature as an alternative method that attracts attention in terms of both its simplicity and simulation accuracy [8]. Kirchhoff approach provides opportunity to model surface scattering in a simplified but sufficiently accurate way. It is used especially in the simulation of surfaces with Lambertian characteristics and allows the analysis of acoustic energy distribution on the basis of angular density [8]. However, the application of this method with different diffuser geometries and its comparative analysis with models developed in the MATLAB environment have not been sufficiently addressed in the literature.

On the other hand, studies comparing the angular reflection performances of Lambertian scatterers and band-limited diffuser surfaces are quite limited. In their studies on the interaction of sound waves with surfaces, Yu et al. [1] and Shi et al. [7] stated that such surfaces exhibit different reflective behaviors at certain frequencies, but they did not analyze the performance differences according to the incidence angle.

In conclusion, the current literature includes valuable studies on frequency response of acoustic diffusers, effects of surface parameters and simulation techniques. However, there is a lack of a holistic study that numerically investigates the relationship between surface geometry and incidence angle, compares Lambertian and band-limited diffusers and simulates this process with Kirchhoff approach in MATLAB. This research aims to fill this gap and examines the acoustic performance of different diffuser geometries from a multi-angle perspective.

This study aims to fill the gap mentioned above. In particular, it is aimed to analyze the reflection characteristics of two different scattering surface types, Lambertian and band-limited diffusers, at different incidence angles using the Kirchhoff approach. These analyses are modeled in MATLAB and supported by simulations; the effect of surface geometry (slope, width, etc.) on performance is numerically evaluated.

The original contributions of this study are as follows:

- It was investigated how the reflection intensities of two diffuser types change depending on the incident angle with the Kirchhoff approach.
- The effect of geometric parameters (e.g., surface slope and indentation width) on the reflection profile was analyzed.
- The accuracy of the model was evaluated by comparing the simulation results with the ideal Lambertian distribution.
- In the light of the findings, design recommendations were presented regarding which diffuser type performs more effectively under which conditions.

In this respect, the study contributes to engineering approaches for the optimization of acoustic diffusers and also brings a new perspective to the literature for a more controlled and efficient dissipation of acoustic energy.

2. Materials and methods

We first assume that circularly polarized light strikes a one-dimensional, random, rough surface that is a perfect conductor. This level is defined by the equation $x_3 = \zeta(x_1)$. The region $x_3 > \zeta(x_1)$ is vacuum, the region $x_3 < \zeta(x_1)$ is a perfect conductor, and the collision plane is the plane x_1, x_3. It is assumed that the surface characteristic function is a stochastic process that need not be static, and the surface is illuminated from the vacuum region. The average derivative of the reflection coefficient $\langle {}^{\partial R_s}/_{\theta_s} \rangle$ is defined as

$\langle\langle {}^{\partial R_s}/_{\theta_s}\rangle\, d\theta_s$, where the parentheses symbol indicates the average over a series of realizations, and the fraction of the time-averaged total flux incident on the surface, distributed as a boundary along the angular distance $(\theta_s, \theta_s + d\theta_s)$, will be $d\theta_s \to 0$. In the limit of geometric optics, the Kirchhoff approximation is written in Equation 1:

$$\langle\frac{\partial R_s}{\theta_s}\rangle = \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{1}{\cos\theta_0}\left[\frac{1+\cos(\theta_0+\theta_s)}{\cos\ \theta_0+\cos(\theta_s)}\right]^2$$
$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} du \exp\{i(q-k)u\} \times (\exp\{-iau\zeta'(x_1)\} \tag{1}$$

In Equation 1, L_1 is the rays with incidence and scattering angles θ_0 and θ_s covered by the random surface with axis length x_1.

$$a = \left(\frac{\omega}{c}\right)(\cos\ \theta_0 + \cos\ \theta_s),$$
$$q = \frac{\omega}{c}\sin\theta_s, \tag{2}$$
$$k = \frac{\omega}{c}\sin\ \theta_0$$

To design the surface characteristic function, we consider the following.

$$\zeta'(x_1) = \sum_{l=-\infty}^{\infty} c_l d(x_1 - 2lb) \tag{3}$$

That is, c_l independent positive random deviations. The function $S(x_1)$ is defined as follows:

$$s(x_1) = \left\{\begin{matrix} 0 & x_1 \le -(m+1)b \\ -(m+1)bh - hx_1 & -(m+1)b < x_1 < -(m)b \\ -bh & -(m)b \le x_1 \le (m)b \\ -(m+1)bh + hx_1 & (m)b \le x_1 \le (m-1)b \\ 0 & (m-1)b \le x_1 \end{matrix}\right\} \tag{4}$$

(m) is a positive number and b represents the length of the feature. Such trapezoidal grooves are also shown in Figure 1.

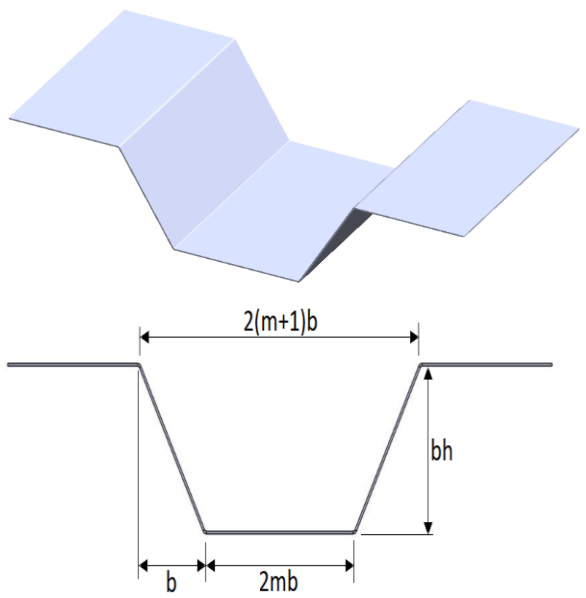

Figure 1 Equation of $S(x_1)$ S(x1)

Since the random deviations c_l are positive, the density functions with probability $f(\gamma) = \langle\delta(\gamma - c_l)\rangle$ are different from zero only for positive values. When the surface characteristic function is in the form of equations (3) and (4), the reflection coefficient of the mean derivative of equation (5) is obtained in the following form.

$$\left\langle\frac{\partial R_s}{\partial\theta_s}\right\rangle = \frac{1}{2h}\frac{[1+\cos(\theta_5+\theta_0)]^2}{\cos\ \theta_0(\cos\ \theta_s + \cos\ \theta_0)^3}$$
$$\times \left[f\left(\frac{\sin\ \theta_0 - \sin\ \theta_s}{h(\cos\ \theta_0 + \cos\ \theta_g)}\right) \right. \tag{5}$$
$$\left. + f\left(\frac{\sin\ \theta_s - \sin\ \theta_0}{h(\cos\ \theta_0 + \cos\ \theta_3)}\right)\right]$$

The Lambertian scatterer assuming normal collision $\theta_0 = 0$ can be written using equation (6).

$$\left\langle\frac{\partial R_s}{\partial\theta_s}\right\rangle = \frac{1}{4h}\left(1+\tan^2\frac{\theta_s}{2}\right)\times\left[f\left(\frac{-1}{h}\tan\frac{\theta_s}{2}\right) + f\left(\frac{1}{h}\tan\frac{\theta_s}{2}\right)\right] \tag{6}$$

Since the scattering surface is Lambertian, equation (7) should finally be as follows:

$$\langle\frac{\partial R_s}{\partial\theta_s}\rangle = \frac{1}{2}\cos\ \theta_s \tag{7}$$

When the variables $\tan\frac{\theta_s}{2} = \gamma h$ and $0 \le \gamma h \le 1$ are changed to $\frac{1}{2}\cos\ \theta_s = 1/2\left(1 - \frac{\gamma^2 h^2}{1} + \gamma^2 h^2\right)$, equation (8) is obtained.

$$f(-\gamma) + f(\gamma) = 2h\frac{1-\gamma^2h^2}{(1+\gamma^2h^2)^2} \tag{8}$$

From this relation we can obtain the probability density function for the Lambertian surface.

$$f(\gamma) = 2h\frac{1-\gamma^2h^2}{(1+\gamma^2h^2)^2}\theta\left(\frac{1}{h}-\gamma\right)\theta(\gamma) \tag{9}$$

In a band limited uniform diffuser, the dispersion is uniformly distributed over the angular range $-\theta_m < \theta_s - \theta_0 < \theta_m$. As a result $({}^{\partial R_s}/_{\partial\theta_s})$ should be as equation 11.

$$\langle\frac{\partial R_s}{\partial\theta_s}\rangle = \text{Krect}\left(\frac{\theta_S-\theta_0}{\theta_m}\right) \tag{10}$$

Given the constraint that f(γ) is nonzero for (γ>0) and thus each (c_l) is a positive number, equation 11 is obtained:

$$f(\gamma) = \text{rect}\left(\gamma - \frac{1}{2}\right) \tag{11}$$

$(f(\gamma))$ With this choice for , equation 12 is obtained.

$$\langle\frac{\partial R_s}{\partial\theta_s}\rangle = \frac{1}{4h}\left[\text{rect}\left(-\frac{\theta_s}{2h}-\frac{1}{2}\right) + \frac{1}{2}\text{rect}\left(\frac{\theta_s}{2h}-\frac{1}{2}\right)\right] = \frac{1}{4h}\text{rect}\left(\frac{\theta_s}{4h}\right) \tag{12}$$

3. Results

For a surface to act as a Lambertian diffuser, parameter (b) must be large enough, i.e., the grooves must be wide. Parameter (h) represents the slope of the surface. Figure 2 shows the Lambertian reflection from the surface.

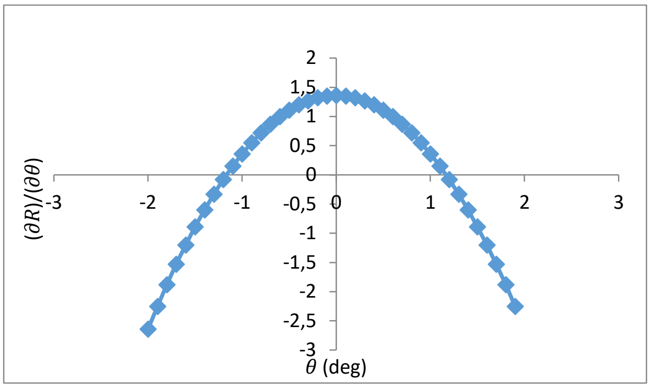

Figure 2 Lambertian surface reflectance with characteristic function equation 1 and density function equation 10

Figure 2 shows the reflection properties of a Lambertian surface calculated using the characteristic function equation 1 and the density function equation 10. The graph shows that the reflection intensity ($\frac{\partial R}{\partial \theta}$) is distributed symmetrically and has an angularly smooth profile. This is a result that is in accordance with the theoretical properties of Lambertian surfaces, confirming that the energy distribution is evenly distributed around the surface normal. Furthermore, this graph shows that the design parameters of the surface are consistent with the theoretical models. As seen in Figure 3, the slope increases with the increase in (h) and the reflection intensity decreases.

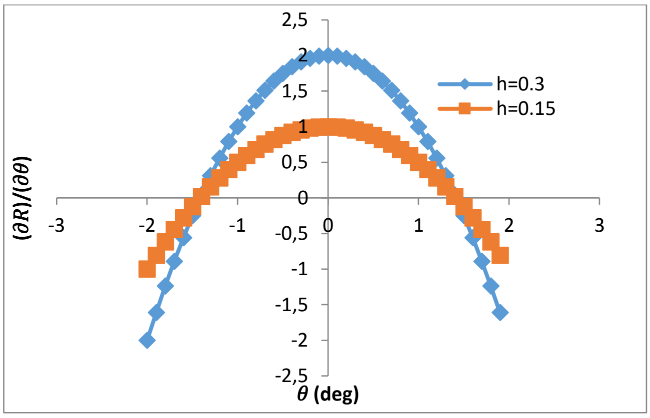

Figure 3 The reflectivity of an ideal Lambertian surface for two different slopes decreases as the slope increases.

Figure 3 shows the angular distribution of the reflection intensity ($\frac{\partial R}{\partial \theta}$) of an ideal Lambertian surface for two different surface inclinations (h = 0.3) and (h = 0.15). It is observed from the graph that the reflection intensity decreases and the distribution widens figure with the increase in inclination. While the reflection shows a more pronounced inclination in the case of higher inclination (h = 0.3), it is understood that the reflection is more homogeneous and the energy is spread over a wide angular range for lower inclination (h = 0.15). This situation reveals that the surface inclination affects the deviations from the Lambertian character by controlling the reflection properties and is an important design parameter to optimize the reflection performance.

Simulation results show that the Kirchhoff method is only valid for a zero-degree incidence angle. As seen in Figure 4, the reflection moves out of the Lambertian mode as the incidence angle increases.

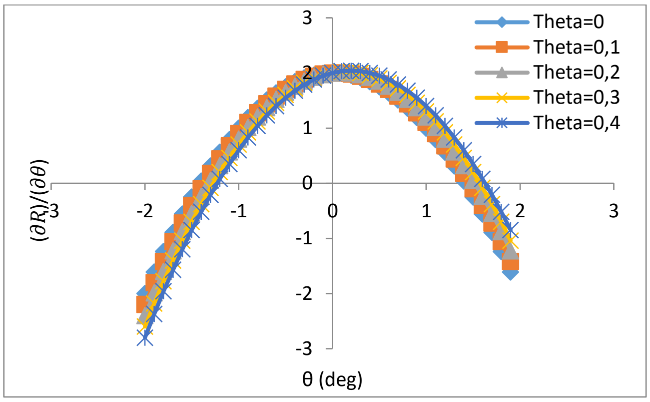

Figure 4 Reflection for different angles of incidence

Figure 4 shows the angular distribution of reflection intensity ($\frac{\partial R}{\partial \theta}$) for different incidence angles (θ_0). The graph reveals that for small incidence angles ($\theta_0 = 0$), the reflection is close to the ideal Lambertian mode, but deviates from this mode as the incidence angle increases ($\theta_0 = 0.1, 0.2, 0.3, 0.4$). This deviation causes the reflection intensity to take on an asymmetric structure and the maximum intensity to be concentrated at different angles. It is observed that for high incidence angles, the reflection is less homogeneously distributed and the diffuser performance is concentrated in a certain angular direction. This shows that the design parameters and the diffuser geometry significantly affect the reflection properties depending on the incidence angle.

Figure 5 and Figure 6 show the reflection from a band-limited uniform diffuser for two different incident angles. Figure 5 shows the angular intensity ($\frac{\partial R_s}{\partial \theta_s}$) of the reflection from a band-limited uniform diffuser for the incident angle ($\theta_0 = 0$). The graph reveals that the energy reflected by the diffuser is concentrated and uniformly distributed within a certain angle range. As can be seen from the figure, the reflection energy is approximately homogeneously distributed between (-10) and (+10) degrees.

The plateau form in this region represents the diffuser's success in distributing the reflected energy equally in this angle range. In the case of low incident angle (θ0 = 0), the reflection exhibits a symmetrical behavior and it is proven that the design of the diffuser provides a uniform acoustic energy distribution. Outside the region where the reflection plateaus ($\theta < -10$ and $\theta > 10$), the reflection intensity approaches zero, indicating that the energy does not shift to the outer regions. This indicates that the diffuser is designed in accordance with the band-limited operating principle and that the reflection only occurs in the targeted angle range. Figure 5 successfully demonstrates the design's compatibility with theoretical models and the diffuser's ability to provide a symmetrical reflection performance.

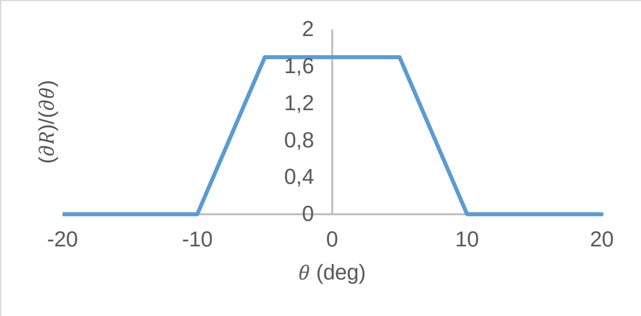

Figure 5 Regular reflection for $\theta_0 = 0$.

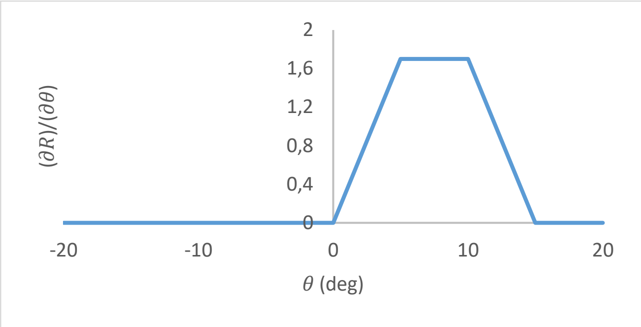

Figure 6 Regular reflection for θ_0 =10.

Figure 6 shows the angular distribution of the reflection from a band-limited uniform diffuser for the incident angle (θ_0 = 10). The graph examines the angular intensity of the reflection ($\frac{\partial R_s}{\partial \theta_s}$) as a function of the angle θ. As can be seen from the figure, the reflection is limited to a certain angle range and the energy is concentrated approximately between (0 degrees and 10 degrees). This situation shows that the diffuser focuses the energy in a narrow angle band by providing a uniform reflection and thus provides a controllable acoustic performance. The region where it rises and plateaus clearly in Figure 6 represents the angle range where the diffuser provides maximum reflection efficiency. After this angle, the reflection rapidly approaches zero, revealing that the energy loss is at a minimum level at other angles. This result shows that the diffuser design is optimized to provide band-limited uniform reflection and that such reflection properties offer a significant advantage in environments where the acoustic energy distribution needs to be homogenized. The figure proves the accuracy of both the design process and the theoretical analysis.

4. Conclusion

In this study, we simulated two different types of two-dimensional scattering surfaces, namely Lambertian surface and band-limited uniform scattering surface, using the Kirchhoff approximation method. The simulation results for the Lambertian surface show that in order to have a suitable reflectance value, the incidence angle should be perpendicular to the surface and the grooves should be wide enough to minimize the shadowing and covering effect. In the Kirchhoff method, the surface roughness does not depend on the wavelength, i.e., a surface can operate at all wavelengths. This study has shown that Matlab software is an effective tool in the design of two-dimensional diffuser surfaces. Simulation results have shown that the surface geometry plays a decisive role in the distribution of acoustic energy and that sound can be distributed homogeneously with the right design parameters. In particular, frequency response analyses have proven that the designed diffusers exhibit effective performance in different acoustic environment conditions. However, it has been stated that further experimental studies and tests with different surface materials are required to ensure that theoretical models are compatible with real-world applications. These results are a valuable guide for engineering applications aiming at acoustic performance improvement.

Limitations

First of all, the modeling has been carried out only at the theoretical and numerical level; experimental verification has not been done. The Kirchhoff approach used may be insufficient to fully represent high surface roughness or complex structures. In addition, material properties are assumed to be fixed; frequency-dependent acoustic responses have not been systematically evaluated.

Future Research

Future research should be expanded to include comparison of the model with experimental data, applications with different types of materials, and three-dimensional surface structures. In addition, approaches such as optimization of diffusers operating in wide frequency bands and artificial intelligence-supported geometry design can provide higher accuracy and efficiency in this area. Studies in this direction will further advance the spatial control of acoustic performance.

Author contributions

M. Mustafaoğlu: Writing, Literature search, analysis, methodology, Investigation, analysis

A. Razmi: Supervision, literature search, writing manuscript

E. Mustafaoğlu: Writing, Literature search

Conflict of Interest

The author has no conflicts of interest to declare.

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